Proceedings of 2006 ISFA
2006 International Symposium on Flexible Automation
Osaka, Japan, July 10-12, 2006

0090-a(S)

DYNAMICS AND CONTROL OF A SPHERICAL ROLLING ROBOT EQUIPPED WITH A GYRO

Takateru Urakubo[1*], Satoshi Maekawa[2], Hisashi Tamaki[1], Yukio Tada[1], Toshiaki Otani[3], Hiroshi Kobayashi[3]

1. Faculty of Engineering, Kobe University
2. National Institute of Information and Communication Technology
3. Graduate School of Science and Technology, Kobe University

Email: urakubo@cs.kobe-u.ac.jp

ABSTRACT

In this paper, we propose a new driving mechanism for a spherical rolling robot and investigate the dynamic characteristics of the robot by theoretical analysis and numerical simulations. The spherical robot has a momentum wheel(gyro) that rotates at a large velocity inside the robot, and its mechanism may be expected to provide the driving force efficiently. However, the dynamics of the robot is very complex because of the angular momentum of the gyro. We derive the equations of motion for the spherical robot that include the effects of frictional forces, and perform numerical simulations in order to examine the behavior of the robot.

INTRODUCTION

Many studies on autonomous mobile robots such as a wheeled mobile robot and a biped walking robot have been carried out so far. In recent years, development of another kind of mobile robot, a spherical rolling robot, has attracted the interest of many researchers. A spherical robot rolls and moves on the floor by using some actuators located in its inside, and would be practically useful because it can achieve omnidirectional motion. The driving mechanisms of the robot that have already been proposed are grouped into several types according to the actuators; reaction wheel type(Bhattacharya and Agrawal, 2000), moving mass type(Javadi and Mojabi, 2004, Alves and Dias, 2003), and so on. We are now developing a spherical robot that has a new type of driving mechanism equipped with a gyro(Figs. 1 and 2). The gyro is rotating with a large velocity in the inside of the robot, and the mechanism is designed so that the center of mass of the robot lies at the geometric center of the sphere. By using three motors located inside the robot, some of the angular momentum of the gyro is transferred to an outer spherical shell of the robot, and the spherical robot moves on the flat floor. Although the mechanism may be expected to provide the driving force efficiently, the dynamics of the robot is very complicated because of the angular momentum that the gyro has.

In this paper, we investigate the dynamic characteristics of the robot by theoretical analysis and numerical simulations. The equations of motion for the spherical robot are derived by modeling it as a multi body system. These equations include the effects of friction at the contact point between the outer shell and the floor and friction at the internal mechanism. Under some assumptions, the motion equations show that the angular momentum of the robot about the contact point is conserved and that nutation of an inner subsystem may be caused like a dual-spin satellite. We design some simple controllers to control the translational motion of the outer shell, and examine the behavior of the controlled system by numerical simulations. If there is a friction at the internal mechanism, nutation of the inner subsystem may be quickly damped. If the mass center of the robot is a short distance away from the center of the sphere, precession of the system may be caused by the gravitational force.

MODEL OF THE SPHERICAL ROBOT

Prototype of the Robot

Figure 1 shows a photograph of the spherical mobile robot under development. The radius of sphere is 15[cm], the weight of the robot is about 4.9[kg], and power supply, sensor and etc. are located in the robot so that it can move autonomously. The spherical robot is composed of four bodies, gyro, gimbal, gyro

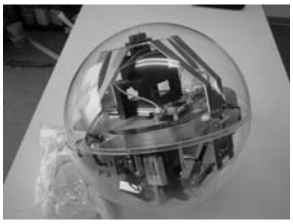

Figure 1. Prototype of a spherical robot

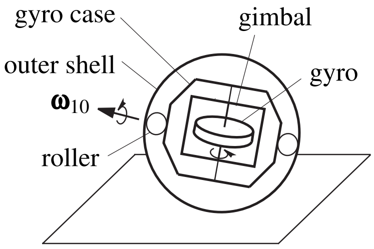

Figure 2. Schematic model of the robot

case and outer spherical shell(Fig. 2). The hardware of the robot is designed so that the mass center of the system would coincide with the geometric center of the sphere. Two motors are located at the rollers attached to the gyro case in order to generate the driving torques about the axes perpendicular to the rotation axis of the gyro, and one motor is located in the gimbal in order to generate the driving torque about the rotation axis of the gyro. By using these motors, some of the angular momentum of the gyro is transferred to the outer shell, and the robot moves on a horizontal surface. At the initial point, the direction of the rotation axis of the gyro is set to be perpendicular to the surface.

Equations of Motion

We consider the motion of the spherical robot that rolls on a horizontal surface. The equations of motion are derived by modeling it as a multi body system composed of three rigid bodies. It is assumed that the gyro case and the gimbal are regarded as single rigid body for simplicity, and it is called *gyro case* again. The three bodies, outer shell, gyro case and gyro, are labeled as body 1, 2 and 3 respectively. We introduce a set of unit vectors $\{\boldsymbol{a}^{(0)}\} = \{\boldsymbol{a}_1^{(0)}\boldsymbol{a}_2^{(0)}\boldsymbol{a}_3^{(0)}\}$ fixed in an inertial space and a set of unit vectors $\{\boldsymbol{a}^{(i)}\} = \{\boldsymbol{a}_1^{(i)}\boldsymbol{a}_2^{(i)}\boldsymbol{a}_3^{(i)}\}$ fixed in body i ($i = 1,2,3$). The origin of $\{\boldsymbol{a}^{(i)}\}$ is the mass center of body i, and the directions of $\boldsymbol{a}_j^{(i)}$ are along the principal axes of inertia of body i ($j = 1,2,3$). And $\boldsymbol{a}_3^{(0)}$ is chosen so that it is along the vertical direction.

We introduce the following vectors and matrix.

$\boldsymbol{\omega}_{ij}$: angular velocity of $\{\boldsymbol{a}^{(i)}\}$ with respect to $\{\boldsymbol{a}^{(j)}\}$

$\boldsymbol{r}_0$: position vector from the origin of $\{\boldsymbol{a}^{(0)}\}$ to the geometric center of the sphere of the outer shell, $r_0^{(0)} = [x,y,z]^T$

$\boldsymbol{R}_i$: position vector from the geometric center of the sphere to the origin of $\{\boldsymbol{a}^{(i)}\}$ ($i = 1,2,3$)

$\boldsymbol{r}_s$: position vector from the geometric center of the sphere to the contact point between the outer shell and the horizontal surface, $r_s^{(0)} = [0,0,-r]^T$

$A^{(i,j)}$: a coordinate transform matrix from $\{\boldsymbol{a}^{(j)}\}$ to $\{\boldsymbol{a}^{(i)}\}$

where r is a radius of the sphere, and, for a vector $\boldsymbol{b}$, we denote the expression of the vector $\boldsymbol{b}$ in the frame $\{\boldsymbol{a}^{(i)}\}$ as $b^{(i)}$. The mass and inertia of each body are denoted as follows.

m_i : mass of the body i

$\boldsymbol{J}_i$: inertia matrix of the body i about the origin of $\{\boldsymbol{a}^{(i)}\}$, J_i is the expression of $\boldsymbol{J}_i$ in the frame $\{\boldsymbol{a}^{(i)}\}$

The orientation of $\{\boldsymbol{a}^{(1)}\}$ with respect to $\{\boldsymbol{a}^{(0)}\}$ and the one of $\{\boldsymbol{a}^{(2)}\}$ with respect to $\{\boldsymbol{a}^{(1)}\}$ are denoted by Euler parameters, $p = [p_0,p_1,p_2,p_3]^T$ and $q = [q_0,q_1,q_2,q_3]^T$ respectively. The Euler parameters p and q always satisfy the relation of $\sum_{i=0}^{3} p_i^2 = \sum_{i=0}^{3} q_i^2 = 1$, and their time derivatives are calculated as follows(Schaub and Junkins, 2003).

$$\dot{p} = \frac{1}{2}Q(p)\,\omega_{10}^{(1)}\ ,\ \dot{q} = \frac{1}{2}Q(q)\,\omega_{21}^{(2)}\ ,\ Q(s) = \begin{bmatrix} -s_1 & -s_2 & -s_3 \\ s_0 & -s_3 & s_2 \\ s_3 & s_0 & -s_1 \\ -s_2 & s_1 & s_0 \end{bmatrix}$$

The orientation of $\{\boldsymbol{a}^{(3)}\}$ with respect to $\{\boldsymbol{a}^{(2)}\}$ is expressed by the rotation angle θ of the gyro.

From the above expressions, the state of the system is denoted by $\xi = [r_0^{(0)T}, p^T, q^T, \theta]^T$. The time derivative of ξ can be expressed as

$$\dot{\xi} = \begin{bmatrix} I_3 & & & \\ & \frac{1}{2}Q(p) & & \\ & & \frac{1}{2}Q(q) & \\ & & & 1 \end{bmatrix} \eta \equiv U\eta\ , \tag{1}$$

where I_3 is a 3×3 unit matrix and $\eta = [\dot{r}_0^{(0)T}, \omega_{10}^{(1)}, \omega_{21}^{(2)}, \dot{\theta}]^T$. Since the spherical robot rolls on a horizontal surface, the outer shell always touches the surface. Therefore, the following holonomic constraint holds: $\Phi = [0,0,1](r_0^{(0)} + r_s^{(0)}) = 0$. Under this constraint, the equations of motion for the spherical robot are summarized in the following form.

$$\dot{L} + \Omega L + VP = N + F + \lambda_\Phi U^T(\partial\Phi/\partial\xi)^T + \tau \tag{2}$$

In Eq.(2), L is the generalized momentum corresponding to the variable η, N and F are the generalized forces including gravitational force and frictional force respectively, λ_Φ is the normal force at the contact point between the sphere and the horizontal surface, and τ is a term depending on the input torque $\boldsymbol{\tau}_2$ that acts on the gyro case from the outer shell.

Friction Models

We introduce some friction models in order to determine the generalized force F in Eq.(2). Three kinds of frictions at the contact point between the outer shell and the floor and a friction between the outer shell and the gyro case are considered.

a) sliding friction

If the spherical robot rolls on a horizontal surface without slipping, the motion is subject to the following nonholonomic constraint: $\Psi = \dot{r}_0^{(0)} - r_s^{(0)} \times \omega_{10}^{(0)} = 0$, where Ψ is a velocity vector of the contact point expressed in the frame $\{\boldsymbol{a}^{(0)}\}$. Then, by the Lagrange undetermined multiplier method, F includes a term expressed as $F_s = \lambda_\Psi(\partial\Psi/\partial\eta)^T$. Here λ_Ψ is a sliding frictional force under no-slip condition, and it is assumed that $\|\lambda_\Psi\| \le \mu\lambda_\Phi$ where μ is a static friction coefficient. If $\|\lambda_\Psi\| > \mu\lambda_\Phi$, the slip of the outer shell occurs. Then, we assume that the following dynamic friction $k^{(0)}$ acts at the contact point : $k^{(0)} = -\mu'\lambda_\Phi\Psi/\|\Psi\|$, where μ' is a dynamic friction coefficient.

When the outer shell slips on the floor, the term F_s is calculated from the dynamic friction $k^{(0)}$ and has the form $F_s = F_s(k^{(0)})$.

b) rolling friction

When the outer shell rolls on a horizontal surface, it may be assumed that the following rolling friction acts at the contact point due to small deformations of the outer shell and the surface: $M_r^{(0)} = (\mu_r/\|\dot{r}_0^{(0)}\|)\cdot\dot{r}_0^{(0)} \times [0,0,\lambda_\Phi]^T$, where μ_r is a positive constant. Then, a term $F_r(M_r^{(0)})$ is added to the generalized force F.

c) frictional torque at the contact point about the vertical axis

When the outer shell is rotating about an axis perpendicular to the horizontal surface, a frictional torque to damp the rotation may act on the outer shell. We assume that it has the following form: $M_z^{(0)} = -\mu_z\lambda_\Phi[0,0,1]\omega_{10}^{(0)}$, where μ_z is a positive constant. Then, a term $F_z(M_z^{(0)})$ is added to the generalized force F.

d) frictional torque at the internal mechanism

The actuators between the outer shell and the gyro case generate the driving torque $\boldsymbol{\tau}_2$. The driving mechanism includes motors, gears, belts, etc., and some frictions exist between the outer shell and the gyro case. We assume that they can be expressed by $M_i^{(2)} = -\mu_i\omega_{21}^{(2)}$, where μ_i is a positive constant. Then, a term $F_i(M_i^{(2)})$ is added to the generalized force F.

DYNAMIC CHARACTERISTICS OF IDEAL ROBOT

In this section, we consider an ideal robot that satisfies the following assumptions.

Assumptions

1. The center of mass of the spherical robot lies exactly at the geometric center of the sphere. That is, $\boldsymbol{R}_1 = 0$ and $m_2\boldsymbol{R}_2 + m_3\boldsymbol{R}_3 = 0$.
2. Any frictional forces other than sliding frictional force do not work. Therefore, $F = F_s$.
3. The rotation axis of the gyro is parallel to one of the principal axes of inertia of the gyro case.
4. The outer shell is completely spherically-symmetric. Therefore, $J_1 = \mathrm{diag}\{j, j, j\}$.

Under the above assumptions, the angular momentum of the spherical robot about the contact point is conserved.

$$\boldsymbol{H} = \sum_{i=1}^{3} \boldsymbol{J}_i\boldsymbol{\omega}_{i0} - \boldsymbol{r}_s \times (m_1+m_2+m_3)\dot{\boldsymbol{r}}_0 = \text{const.} \qquad (3)$$

Furthermore, the equations of motion of a subsystem composed of the gyro case and the gyro can be expressed as follows.

$$\sum_{i=2}^{3} \left(\dot{\boldsymbol{J}}_i\boldsymbol{\omega}_{i0} + \boldsymbol{J}_i\dot{\boldsymbol{\omega}}_{i0}\right) = \boldsymbol{\tau}_2 \ , \ \ \boldsymbol{a}_3^{(3)T}\dot{\boldsymbol{\omega}}_{30} = 0 \qquad (4)$$

Equation (4) is the same as the one for a dual-spin satellite that an external force $\boldsymbol{\tau}_2$ acts on. Precession and nutation may be caused by the angular momentum that the gyro has. Nutation period T can be approximately obtained from Eq.(4) as follows.

$$T = 2\pi\sqrt{(J_{2x}+J_{3xy})(J_{2y}+J_{3xy})}/(J_{3z}|\dot{\theta}|) \ , \qquad (5)$$

where $J_2 = \mathrm{diag}\{J_{2x}, J_{2y}, J_{2z}\}$ and $J_3 = \mathrm{diag}\{J_{3xy}, J_{3xy}, J_{3z}\}$.

NUMERICAL SIMULATIONS

We carried out many numerical simulations by using some simple controllers and various sets of physical parameters. For the sake of brevity, this section provides only the results in three cases that represent characteristic behaviors of the system.

Controllers

We consider two types of control inputs, an impulse input and a feedback input.

• **Controller 1** (impulse input)

$$\tau_2^{(2)} = \begin{cases} [0, 1.0, 0]^T \ [\mathrm{N{\cdot}m}] & \text{for } 0 \le t < 8.0\times10^{-2}[\mathrm{s}] \\ [0,0,0]^T \ [\mathrm{N{\cdot}m}] & \text{for } t \ge 8.0\times10^{-2}[\mathrm{s}] \end{cases} \qquad (6)$$

• **Controller 2** (position control)

To make the robot move in a straight line with constant velocity and change the position of the robot, the following simple feedback controller is designed.

$$\tau_2^{(2)} = K_1A^{(2,0)}(\omega_{10}^{(0)} - \omega_{10d}^{(0)}) - K_2\mathrm{diag}\{1,1,0\}\omega_{20}^{(2)} \ , \qquad (7)$$

where K_1 and K_2 are positive constants and $\omega_{10d}^{(0)}$ is the desired angular velocity of the outer shell. K_1 and K_2 are set to 1.0 and 1.0 respectively, and $\omega_{10d}^{(0)}$ is chosen as

$$\omega_{10d}^{(0)} = \begin{cases} [0, 1.0, 0]^T & \text{for } 0 \le t < 5.0[\mathrm{s}] \\ [0,0,0]^T & \text{for } t \ge 5.0[\mathrm{s}] \end{cases} \ . \qquad (8)$$

The second term in the right-hand side of Eq.(7) is added in order to damp nutation of the subsystem.

Physical parameters

In numerical simulations, we choose the mass and inertia of each body as follows. $m_1 = 1.0$[kg], $m_2 = 2.8$[kg], $m_3 = 1.0$[kg], $J_1 = \mathrm{diag}\{0.015, 0.015, 0.015\}[\mathrm{kg{\cdot}m^2}]$, $J_2 = \mathrm{diag}\{0.018, 0.018, 0.020\}[\mathrm{kg{\cdot}m^2}]$, and $J_3 = \mathrm{diag}\{0.0026, 0.0026, 0.0052\}[\mathrm{kg{\cdot}m^2}]$. The radius of the sphere r is set to 0.15[m]. The center of mass of each body is chosen as in the following two cases.

• **Parameters 1** (assumption 1 is satisfied)
$R_1 = R_2 = R_3 = [0,0,0]^T$[m].

• **Parameters 2** (assumption 1 is not satisfied)
$R_1 = [0,0,0]^T$[m], $R_2 = [0.004, -0.006, -0.007]^T$[m] and $R_3 = [0,0,0.01]^T$[m].

At the initial point, the outer shell and the gyro case are at rest, and the rotational velocity of the gyro $\dot{\theta}$ is set to 100[rad/s]. Therefore, the initial angular momentum of the robot is $H^{(0)} = [0,0,0.52][\mathrm{kg{\cdot}m^2/s}]$. The coefficients of frictional forces are chosen as $\mu = 0.8$, $\mu' = 0.3$, $\mu_r = 3.0\times10^{-4}$[m], $\mu_z = 3.0\times10^{-4}$[m·s], $\mu_i = 0.2[\mathrm{kg\cdot m^2/s}]$.

The simulation results are summarized as follows.

Case 1 (Controller 1, Parameters 1 , $F = F_s$)

In this case, the controller (6) is applied to the ideal spherical robot. After $t = 8.0\times10^{-2}$[s], the outer shell moves with a constant velocity $\dot{r}_0^{(0)} = [-0.098, -1.1\times10^{-4}, 0]^T$[m/s], and nutation of the subsystem is caused as shown in Fig. 3. The period of nutation is about 0.25[s], which coincides with the theoretical result in Eq.(5). The angular momentum $\boldsymbol{H}$ is conserved along the motion as shown in Eq.(3). Moreover, if the generalized force F

includes the term F_i, the nutation disappears rapidly because of the friction between the gyro case and the outer shell.

Case 2 (Controller 2, Parameters 1 , $F = F_s + F_r + F_z + F_i$)

In this case, the controller (7) is applied to a spherical robot that does not satisfy the assumption 2. It is assumed that all frictional forces are added to the motion equations, that is, $F = F_s + F_r + F_z + F_i$. Figure 4 shows that the robot moves along the x direction and the translational motion stops at $(x, y) = (0.59, -0.018)$[m]. However, after that, a rotational motion of the outer shell about $\boldsymbol{a}_3^{(0)}$ still remains and slowly fades out. The angular momentum $\boldsymbol{H}$ is not conserved along the motion because of the frictional torques $M_r^{(0)}$ and $M_z^{(0)}$. Therefore, when the translational motion stops, the rotation axis of the gyro can not return to its initial direction(Fig. 5). The angle η between the rotation axis and $\boldsymbol{a}_3^{(0)}$ that is defined as $\eta = \cos^{-1}(\boldsymbol{a}_3^{(0)T} \boldsymbol{a}_3^{(3)})$ is about 0.14[rad] at $t = 10$[s], and reaches its maximum value of about 0.31[rad] at $t = 5$[s].

Case 3 (Controller 2, Parameters 2 , $F = F_s + F_r + F_z + F_i$)

In this case, the controller (7) is applied to a spherical robot that does not satisfy the assumptions 1 or 2. Figure 6 shows the behavior of the rotation axis of the gyro for $0 \le t \le 70$[s]. Precession of the subsystem is caused like a spinning top by the gravitational force. The angle η reaches about 0.82[rad] at $t = 70$[s]. Because of the precession, the translational motion of the outer shell can not stop and the outer shell moves about near $(x, y) = (0.62, 0.01)$[m]

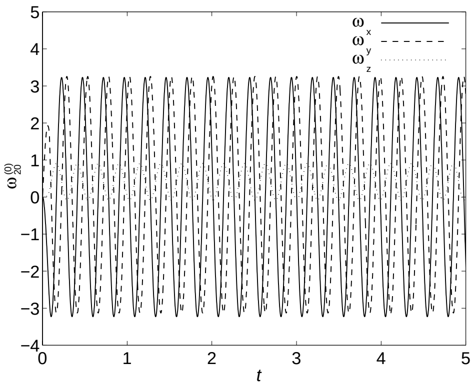

Figure 3. Time histories of $\omega_{20}^{(0)}$ (Case 1)

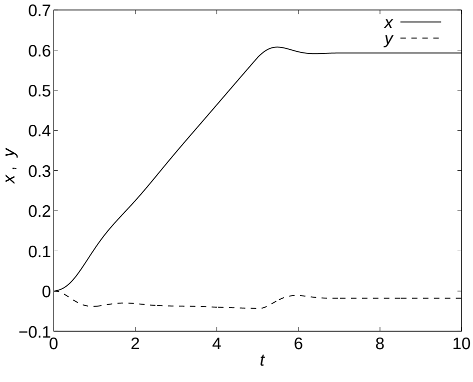

Figure 4. Time histories of x and y (Case 2)

CONCLUSION

In this paper, we derived the equations of motion for a spherical rolling robot equipped with a gyro, and examined the dynamic characteristics of the robot by theoretical analysis and numerical simulations. For an ideal spherical robot that satisfies some assumptions, the angular momentum of the robot is conserved and nutation of the inner mechanism may be caused. However, if there is a frictional torque at the contact point between the outer shell and a horizontal surface, the angular momentum can be changed. In the case where there is a friction between the gyro case and the outer shell, the nutation of the inner mechanism is quickly damped. On the other hand, when the mass center of the system is a short distance away from the center of the sphere, precession of the system may be caused by the gravitational force acting on the system. Motion control of the robot with the precession would be difficult.

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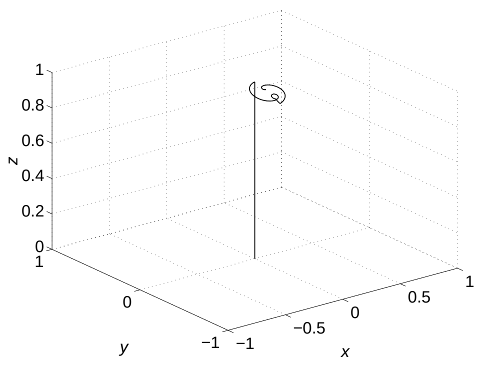

Figure 5. Behavior of the rotation axis of the gyro (Case 2)

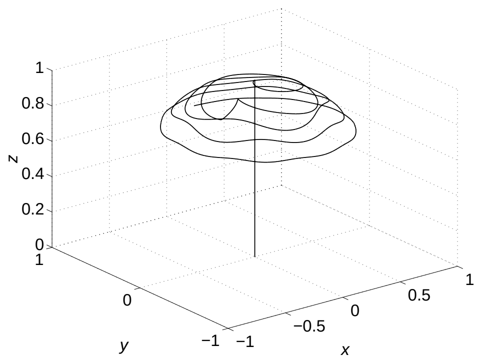

Figure 6. Behavior of the rotation axis of the gyro (Case 3)